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Three flavour QCD from the holographic principle

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Abstract: Building on recent research into five-dimensional holographic models of QCD, we extend this work by including the strange quark with an $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ gauge symmetry in the five-dimensional theory. In addition we deform the naive AdS metric with a single parameter, thereby breaking the conformal symmetry at low energies. The vector and axial vector sectors are studied in detail and both the masses and decay constants are calculated with the additional parameters. It is shown that with a single extra degree of freedom, exceptional agreement with experimental results can be obtained in the light quark sector while the kaon sector of the vector and axial vector octet is found to give around 10% agreement with lattice results. We propose some simple extensions to this work to be taken up in future research.

Keywords: QCD, AdS-CFT Correspondence, Gauge-gravity correspondence.

Contents

1. Introduction

Since 1973 QCD [1] has been the established theory of the strong interactions. Though understood in the perturbative regime, there remains no analytic solution for the phenomenon of confinement for a general Yang-Mills theory. Effective chiral perturbation theory is constructed in order to describe the low energy interactions of matter fields. A systematic method to handle the non-perturbative regime of the strong interaction from first principles has been sought though without success so far. Inspired by the gauge/gravity duality [2 − 4] and the success of flavoured models from ten-dimensional supergravity [5 − 9], several holographic models [10 − 17] have been proposed recently with surprising success. The key point is that although QCD is not a conformal field theory and its string dual is still unknown, it approaches the conformal limit in the ultraviolet regime. Low energy properties such as confinement and chiral symmetry breaking are introduced in these models in a simple manner. The models discussed so far have all included quarks in SU(2) or SU(3) flavour groups, each with an identical mass.

Since the mass differences between the mesons in the same octet are small compared with the scale $\Lambda_{\rm QCD}$ for the lowest vector and axial-vector mesons, it is interesting to study whether the same successful results of the isovector sector can be achieved after including the third flavour. We will show that indeed this is the case. In this letter we will extend the work of [10] to three flavours without assuming the flavour symmetry and calculate the mass spectrum and decay constants of 1^3S_1 vector mesons and 1^3P_1 axial-vector mesons. We will not discuss the scalar sector and the $\mathrm{U}(1)_A$ problem but will leave these questions for future research.

The metric in the five-dimensional bulk with four-dimensional Poincaré symmetry and a compact extra dimension is

$$ds^2 = \theta(z)\theta(z_{\rm IR} - z)a^2(z)(dx^\mu dx_\mu - dz^2), \tag{1.1}$$

where

$$a^2(z) = \frac{1}{z^2}(1 + \sum_{i=1}^{n} \alpha_n z^{2n}) \qquad \text{and } \theta(z) \text{ is the step function.} \tag{1.2}$$

The 5^{th} dimension corresponds to the energy scale in the four-dimensional theory. The metric has a double pole on the UV boundary ($z = 0$) and is asymptotically AdS_5 in the $z \to 0$ limit. The IR boundary $Z_{\rm IR} \sim O(\Lambda_{\rm QCD}^{-1})$ is put in by hand in this model in order to introduce the mass gap and can be interpreted as the position of an IR D-brane. In this work we will consider both the simplest case, that is, the AdS bulk such that $a(z) = \frac{1}{z}$ and the case where only α_1 is non-zero in the warp factor $a(z)$. Since the AdS background is dual to $\mathcal{N} = 4$ SYM, considering the deformed metric is natural for a model builder as QCD is not a conformal field theory. In the bulk the gauge group is chosen to be $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$, which is dual to the global chiral group on the UV boundary where QCD behaves approximately like a CFT. In addition to the gauge fields $L_M(z,x)$ and $R_M(z,x)$, a scalar field X transforming as $(3_L, 3_R)$ is also introduced to induce the chiral symmetry breaking. The non-renormalizable five-dimensional action is

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr} \left\{ -\frac{1}{4g_5^2} \left(L_{\rm MN} L^{\rm MN} + R_{\rm MN} R^{\rm MN} \right) + |DX|^2 + 3|X|^2 \right\}, \tag{1.3}$$

where

$$\begin{aligned} D_M X &= \partial_M X - i L_M X + i X R_M, \\ L_M &= L_M^a t^a, \\ L_{\rm MN} &= \partial_M L_N - \partial_N L_M - i[L_M, L_N], \qquad \text{similarly for } R, \end{aligned} \tag{1.4}$$

where $\mathrm{Tr}[t^a, t^b] = \frac{1}{2}\delta^{\rm ab}$. For our interest in this letter, we keep only terms up to quadratic order in fields. The effects of including higher order non-renormalizable terms is equivalent to deforming the metric from the AdS metric, which will also be considered in this work. However, the α_n terms are blind to flavours. The only source which violates $\mathrm{SU}(3)_{\rm flavour}$ symmetry comes from X. We define the vector and axial-vector gauge bosons to be $V_M = \frac{1}{2}(L_M + R_M)$ and $A_M = \frac{1}{2}(L_M - R_M)$ respectively. Following [10], we will choose the axial gauge $V_z = A_z = 0$. According to the holographic recipe [4, 18], the non-normalizable mode of V_μ is dual to the conserved vector current in QCD while the normalizable modes give the five-dimensional extension of the vector meson wavefunctions, similarly for A_μ fields. Also, the mass of the scalar field X is determined by the AdS/CFT correspondence. First, we consider the pure AdS case. After solving the equation of motion for the expectation value of X, the solution is

$$\langle X(z) \rangle = \frac{1}{2}(Mz + \Sigma z^3), \tag{1.5}$$

where

$$M = \lim_{z\to 0} 2\frac{\langle X(z)\rangle}{z} \qquad \text{and} \qquad \Sigma = \frac{2\langle X(z_{\rm IR})\rangle - M z_{\rm IR}}{z_{\rm IR}^3}.$$

M is the quark mass matrix and $\Sigma = \langle \bar{q}q \rangle$, which can be easily shown by matching the four-dimensional effective Lagrangian to the chiral Lagrangian [11]. Σ is responsible for the χSB in the chiral limit and we expect $\Sigma \propto \mathbf{1}$ in this limit. That is, $\Sigma \sim \alpha \mathbf{1} + O(M)$. Instead of considering possible boundary terms which determine the specific forms of M and Σ, we will parametrize the effects of these terms and choose $M = diag(m, m, m_s)$ in the isospin limit for phenomenological interest and $\Sigma = diag(c, c, c_s)$. With this choice, $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ is broken to $\mathrm{SU}(2) \otimes \mathrm{U}(1)$.

Substituting $X = \langle X \rangle e^{i2t^a \pi^a(x,z)}$ back into the action, one can show that the five-dimensional fields V_μ^a and A_μ^a have block-diagonal z-dependent mass matrices M_V and M_A respectively where

$$M_V^2 = \begin{pmatrix} \mathbf{0_{3\times 3}} & 0 & 0 \\ 0 & \frac{1}{4}\left(m - m_s + (c - c_s)z^2\right)^2 z^2 \mathbf{1_{4\times 4}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{1.6}$$

and

$$M_A^2 = \tag{1.7}$$
$$\begin{pmatrix} (m + cz^2)^2 z^2 \mathbf{1_{3\times 3}} & 0 & 0 \\ 0 & \frac{1}{4}\left(m + m_s + (c + c_s)z^2\right)^2 z^2 \mathbf{1_{4\times 4}} & 0 \\ 0 & 0 & \frac{1}{3}\left(2\left(m_s + c_s z^2\right)^2 + \left(m + cz^2\right)^2\right)^2 z^2 \end{pmatrix}.$$

$V_\mu^{1,2,3}$, $V_\mu^{4,5,6,7}$, and V_μ^8 correspond to isovector, isodoublet, and isosinglet vector mesons in the octet of the quark model, similarly for the axial-mesons. Note that the mass of isovector and isosinglet vector mesons, which are related to M_V, are the same and independent of $\langle X \rangle$. In the large N_c limit [19], where Zweig's rule can be shown to be exact, this result suggests the isosinglet in the octet to be the ω meson in the "ideal mixing" limit. The origin of the mass for V_μ^a is not from the Higgs mechanism but only from the explicitly flavour symmetry breaking term. In the $\mathrm{SU}(3)_{\text{flavour}}$ limit, V_μ^a fields are massless. In this limit, all the vector mesons in the same octet have the same mass.

The coupling constant g_5 is determined by matching the high energy expansion for the two-point function, calculated by the holographic recipe, to the same function calculated from the OPE in large N_c limit [20]. The result is [10] $g_5^2 = \frac{12\pi^2}{N_c}$. This result relates the coupling constant N_c in large N_c QCD to the one in its dual model.

The vector meson, V, wavefunction, Φ_V, and the axial meson, A, wavefunction, Φ_A, which are the normalizable modes of four-dimensional Fourier transformed transverse field $V_{\mu\perp}^a$ and $A_{\mu\perp}^a$ respectively, satisfy the following linearized equations of motion (no summation)

$$\left[\partial_z^2 + \partial_z\left(\ln a(z)\right)\partial_z + \left(q^2 - (g_5^2 a(z)^2 M_V^2)_{aa}\right)\right]\Phi_V^a(q, z) = 0, \tag{1.8}$$

and

$$\left[\partial_z^2 + \partial_z\left(\ln a(z)\right)\partial_z + \left(q^2 - (g_5^2 a(z)^2 M_A^2)_{aa}\right)\right]\Phi_A^a(q, z) = 0, \tag{1.9}$$

with boundary conditions $\partial_z \Phi_V^a(q, z_{\mathrm{IR}}) = 0$ and $\Phi_V^a(q, \epsilon) = 0$, similarly for Φ_A^a. In the large N_c limit, the hadron masses can be obtained from the poles of the two-point function. This is equivalent to solving eq. (1.8) and eq. (1.9) with eigenvalues $q^2 = m_V^2$ and $q^2 = m_A^2$ respectively. The decay constants $F_{V,A}$, which are defined by $\langle 0 \mid J_{V,A\mu}^a \mid V^b, A^b \rangle = \delta^{ab} F_{V,A} \epsilon_\mu$ with ϵ_μ the polarization vector, can be shown to be [10]

$$F_{V,A}^2 = \frac{1}{g_5^2}\left(\frac{\Phi''_{V,A}(0)}{N}\right)^2, \tag{1.10}$$

where

$$N = \int_0^{z_{\rm IR}} dz a(z) |\Phi_{V,A}(z)|^2.$$

The masses of pseudoscalar mesons P can be obtained by solving the following equations mixed by the normalizable modes of the fields $\pi^a(x,z)$ and the longitudinal components of A^a_μ, defined to be $A^a_{\mu\|}(x,z) \equiv \partial_\mu \phi^a(x,z)$, at $q^2 = m_P^2$ with boundary conditions $\phi^{a\prime}(z_{\rm IR}) = \pi^{a\prime}(z_{\rm IR}) = \phi^a(0) = \pi^a(0) = 0$ (no summation)

$$\begin{aligned}
&\left(\partial_z^2 + \partial_z \left(\ln a(z)\right) \partial_z\right) \phi^a + g_5^2 a^2(z) \left(M_A^2\right)_{aa} (\pi^a - \phi^a) = 0, \\
&\partial_z \left(a^3(z) \left(M_V^2 + M_A^2\right)_{aa} \partial_z \pi^a\right) \\
&+ a^3(z) q^2 \left(\left(M_V^2 + M_A^2\right)_{aa} \left(\pi^a - \frac{1}{2}\phi^a\right) + \left(M_V^2 - M_A^2\right)_{aa} \frac{1}{2}\phi^a \right) = 0. \qquad (1.11)
\end{aligned}$$

The decay constants of massless pseudoscalar mesons P^a (which is a good approximation for pions) are given by [10]

$$f_{P^a} = -\frac{1}{g_5^2} \frac{\partial_z A^a(0,z)}{z}|_{z=\epsilon}, \qquad (1.12)$$

with $A^a(0,z)$ the solution of eq. (1.9) satisfying $A^{a\prime}(0, z_{\rm IR}) = 0$ and $A^a(0,\epsilon) = 1$.

The results are shown in table 2 and table 3. It is well-known that without flavour symmetry, the 1^3P_1 isodoublet state mixes with 1^1P_1 isodoublet state. The mixing angle θ is defined by parametrizing the physical state $K_1(1273)$ and $K_1(1402)$

$$\begin{aligned}
|K_1(1402)\rangle &= |K_{1A}\rangle \cos\theta - |K_{1B}\rangle \sin\theta, \\
|K_1(1273)\rangle &= |K_{1A}\rangle \sin\theta - |K_{1B}\rangle \cos\theta, \qquad (1.13)
\end{aligned}$$

such that

$$\sin^2\theta = \left(\frac{M^2_{K_1(1402)} - M^2_{K_{1A}}}{M^2_{K_1(1402)} - M^2_{K_1(1273)}} \right).$$

In the numerical analysis we fit the value of the quark mass and condensate to the case of ideal mixing ($\theta = 45^o$).

We now turn to the case where the warp factor is deformed from the AdS case. We consider the case that only α_1 is non-zero. In this case the solution $\langle X(z)\rangle$ is of the form $\langle X(z)\rangle = diag(X_u(z), X_u(z), X_s(z))$ where $X_u(z)$ and $X_s(z)$ approach their AdS limit respectively when $z \to 0$. All the qualitative discussions above still hold and our numerical results are shown in table 3 and table 4. α_1 is expected to be of order $O(g_5^2 m_q^2)$ from matching the two-point function to the same function in the large N_c QCD. Our numerical result for α_1 agrees with this.

2. Numerical analysis

Precisely as in [10] the value of the IR cutoff is calculated using the equation of motion of the ρ meson, which is independent of the quark mass. The value of this cutoff is however a function of the deformation parameter, α_1, which is tuned such that the ρ decay constant

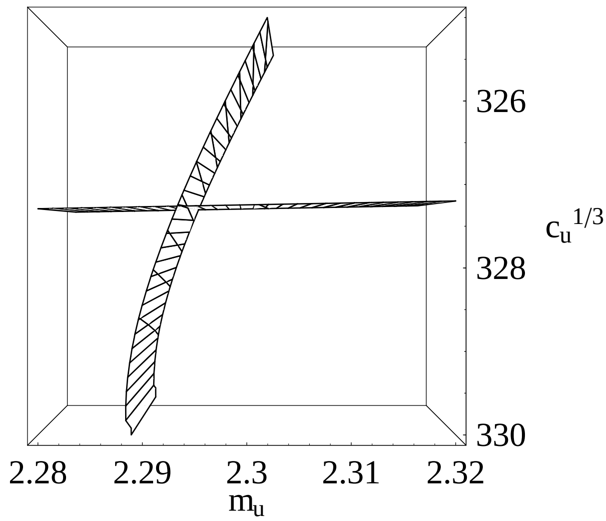

Figure 1: Intersection of the physical contours for m_π and f_π in the $(m, c^{\frac{1}{3}})$ plane.

is closer to the experimental value. Note that by tuning α_1, it is possible to obtain exactly the correct value for this decay constant. In this work a semi-global fit has been used so that the value of the decay constant is good but not exact, such that the predictions of other observables are closer to the experimental values. This method is similar to the global fit used in [10] but here, because we have three more free parameters, a full global fit is unmanageable. This presumably means that a full survey of the parameter space would give even better results than those presented here if such a fit were computable.

Having calculated the IR cutoff, the excited states of the ρ meson are found by studying the higher modes of the appropriate five-dimensional field.

In [10], a perturbative expansion is used to prove the Gell-Mann-Oakes-Renner relation [21]. We show here that an alternative numerical approach can be used to calculate the values of the quark mass m and condensate c. The value of the pion decay constant is a function of the quark mass and condensate. Similarly the boundary behaviour of the five-dimensional field corresponding to the pion is a function of these parameters (with q^2 set to m_π^2 in the pseudoscalar equation of motion). We can plot both of these quantities as a function of the quark mass and condensate. Each plot should have a line in (m, c) space corresponding to the physical values (the correct value of f_π and the correct boundary behaviour respectively). The point at which these lines intersect therefore gives the correct value of m and c to provide the physical pion mass and pion decay constant. These "physical contours" are shown in figure 1 to illustrate that this method produces almost identical values to the perturbative approach [10]. In both the undeformed and deformed AdS cases it can be shown that the GMOR relation holds to order m_q^2 as expected.

The mass of the a_1 is calculated by solving the axial vector equation of motion with the appropriate boundary conditions. It is found that in the case of deformed AdS, the mass

Parameter	Value (MeV) with $\alpha_1 = 225^2$	Value (MeV) with $\alpha_1 = 0$
$Z_{\rm IR}^{-1}$	335.6	322.6
m_u	2.012	2.29
$c_u^{\frac{1}{3}}$	346.25	327.25
m_s	90	100
$c_s^{\frac{1}{3}}$	457.53	232.5

Table 1: Values chosen for the free parameters for both pure AdS_5 and the deformed geometry. Note that in the case of $\alpha_1 \neq 0$, we have performed a semi-global fit over pairs of variables.

Observables	Value (MeV) with $\alpha_1 = 225^2$ (% error)	Value (MeV) with $\alpha_1 = 0$ (% error)
m_π	139.6$*$	139.6$*$
f_π	92.4$*$	92.4$*$
m_{a_1}	1238 (0.24)	1363 (10.4)
$\sqrt{F_{a_1}}$	434.92 (0.44)	486 (12.2)
$m_{K_{1A}}$	1339*	1339*
$\sqrt{F_{K_{1A}}}$	436 ($\sqrt{F_{K_1(1400)}} \sim 454^\dagger$)	450 ($\sqrt{F_{K_1(1400)}} \sim 454^\dagger$)
m_{A_3}	1389	1339
$\sqrt{F_{A_3}}$	430	443

Table 2: Axial sector results for both pure AdS_5 and the deformed geometry using the parameters in table 1. Note that the $\alpha_1 = 0$ results have been calculated using the same numerical techniques as in [10]. The case of ideal mixing ($\theta = 45^o$) has been chosen to fix the strange quark mass and condensate values. Experimental values are chosen as the midpoint of those in [22]. The decay constant of the a_1 is compared with the lattice result [23]. $*$ indicates that this value is used to fix the free parameters, all other values are predictions. † results are taken from [24]. The axial vector meson A_3 corresponds to the isosinglet meson in the octet.

and decay constant values for this axial vector particle are in remarkably close agreement with experiment.

For the kaon sector, we perform a global fit in the space of m_s and c_s by studying the mass of the vector and axial vector K-mesons corresponding to the $K^*(892)$ and K_{1A} mesons. It is possible to get within a few percent of the experimental value (in the case of ideal mixing) though it may be possible by tuning the deformation parameter and the values of m and c correctly to get these masses exact.

This concludes the calculation of the free parameters of the theory. Using these values, the decay constants for the isodoublet and isosinglet sectors are calculated using eq. (1.10).

We have shown by numerical analysis that even without a full global fit, the vector sector is in close agreement with the experimental results. Compared with the results of the QCD sum rules, this holographic action appears to model the vector and axial vector spectrum of the strong force to high precision. A global fit would almost certainly reduce the average error to the order of a percent or two, however as explained above such a calculation would require considerable computing time.

We provide in tables 1, 2 and 3 the values calculated in our analysis.

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Observables	Value (MeV) with $\alpha_1 = 225^2$ (% error)	Value (MeV) with $\alpha_1 = 0$ (% error)
m_ρ	775.8*	775.8*
$\sqrt{F_\rho}$	335.5 (2.8)	329 (4.5)
$m_{\rho'}$	1830	1781
$\sqrt{F_{\rho'}}$	635.38	616.4
m_{K^*}	793 (11)*	799 (10.4)*
$\sqrt{F_{K^*}}$	330.8 ($11^\dagger$)	329 ($11^\dagger$)
m_{V_3}	m_ρ	m_ρ
$\sqrt{F_{V_3}}$	$\sqrt{F_\rho}$	$\sqrt{F_\rho}$

Table 3: Vector sector results for both pure AdS_5 and the deformed geometry. The ρ' is the first excited state of the ρ meson. Experimental values are chosen as the midpoint of those in [22]. $*$ indicates that this value is used to fix the free parameters, all other values are predictions. † results are taken from lattice predictions [25] though these have large uncertainties. The vector meson V_3 corresponds to the isosinglet meson in the octet.

3. Discussion and conclusion

We have extended the work of Erlich et al [10] to include an SU(3) flavour group with a distinct strange quark mass. We also study the effects of a deformation of the metric by the simplest possible factor. This deformation is sufficient to provide a non-conformal region in the effective four-dimensional theory. We show that the effect of adding this extra parameter is enough to give very accurate ($\epsilon_{rms} < 2\%$) masses and decay constants for the light quark sector.

We extend the formalism with the addition of a strange quark and calculate the vector and axial vector masses and decay constants in the s quark sector. The values of the strange quark mass and condensate are tuned to give the K_{1A} and K^* masses to within a few percent of the experimental values (though we assume ideal mixing). This fit is a function of the deformation parameter, α_1, so that with a full global fit it may be possible to lower the average error, which in this analysis is $\sim 7\%$. With the mass and condensate values fixed, we calculate the decay constants which are compared with lattice data providing a good agreement.

Though in the sectors described in this paper an excellent agreement with experiment and lattice is obtained, it is found that the strange pseudoscalar (the kaon) gives a surprisingly poor fit for the parameter values chosen here. It is also possible to tune m_s and c_s such that the kaon mass is correct giving a good fit for the K^* but a poor result for the K_1.

Note that the values of m_u and c have been fixed without reference to the kaon sector. It may well be possible to tune these values such that the kaon sector provides a better fit to the data. In this letter we merely wish to illustrate that these extensions to the work of Erlich et al [10] can provide reasonable fits in the strange quark sector. We leave a full global fit for further calculation.

There are several possible extensions to this work. It would be interesting to study the mass splitting in the isovector spectrum by the addition of different mass u and d quarks and a four-dimensional U(1) gauge interaction. It would also be interesting to study the addition of an operator corresponding to baryons in these models. Several other interesting directions are noted in [10] which should guide us even closer to a true holographic dual of QCD.

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